An experiment on skin effect

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Two experimental apparatus which may be used for laboratory or classroom demonstration of the skin effect phenomenon are described. The first apparatus shows the decrease of a magnetic field as it penetrates a thickness of metal. The second demonstrates the decrease with depth of current in a wire.

I. INTRODUCTION

In many areas of physics there is a shortage of suitable undergraduate laboratory experiments to supplement and demonstrate the theory which is presented in lectures. One such area is electromagnetic theory where the standard experiments only illustrate a small portion of what is usually presented in the theory lectures. The particular aspect of electromagnetic theory which the experimental apparatus described below demonstrates is the phenomenon known as the skin effect. The skin effect arises when the basic electromagnetic equations known as the *general wave equations,* whose standard solution is for transverse electromagnetic waves, are extended to include conductivity. The solution is then modified and includes a term giving an exponential decay of amplitude with distance. The exponential term is of the form

$$\exp\left[-k_0 z(\{[1+(\sigma/\epsilon\omega)^2]^{1/2}-1\}/2)^{1/2}\right],$$

where k_0 is the wave number, z is the distance into the medium, σ is the conductivity, ϵ the permittivity, and ω the angular frequency. In the places where this term is significant it is usually assumed that the conductivity, σ, will be high with respect to $\epsilon\omega$, and this exponential term there reduces to

$$\exp\left[-z/(2/\mu\sigma\omega)^{1/2}\right].$$

The quantity $(2/\mu\sigma\omega)^{1/2}$ is known as the skin depth and is the depth for a $1/e$ decrease of electric or magnetic fields in a good conductor. A further aspect of this subject is associated with the electric currents which flow during penetration of the fields into the conductor and is usually discussed under the heading of eddy current losses.

Two apparatus are described in parts Secs. II and III below which illustrate various aspects of the skin effect. Both parts can easily be completed in a laboratory session and could be used as a demonstration experiment if desired.

II. MAGNETIC SHIELDING

In this part of the experiment the decrease with depth of alternating magnetic fields in a conductor is demonstrated. This same phenomenon is associated with magnetic shielding and eddy currents in transformer laminations, and determines the maximum practical thickness for transformer laminations.

The experimental apparatus for this part of the experiment is very simple and is shown in Fig. 1. For clarity a bisected view of the apparatus is shown. The outer wooden frame, an open cube of approximately 25 cm, with a shelf as shown should not use metallic fasteners, although in our apparatus a few brass screws were used and did not seem to have much adverse effect. The demountable transformers used have a 3 × 3-cm core. If smaller or larger transformers were used the scale of the apparatus would have to be adjusted accordingly.

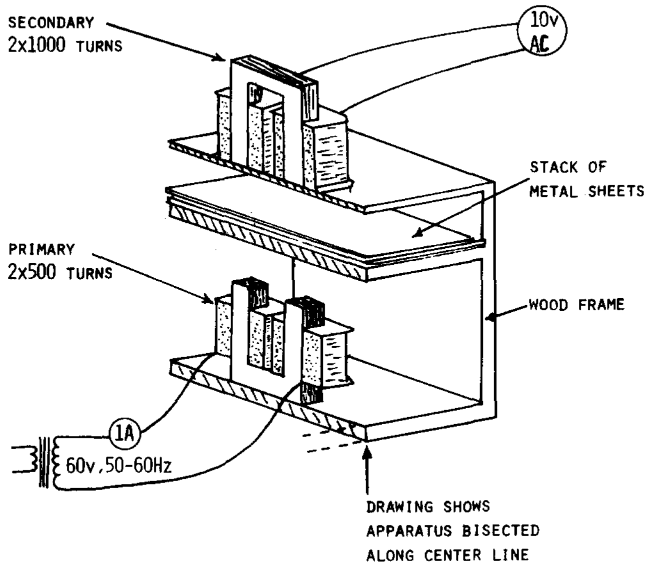

Fig. 1. Apparatus for decrease of magnetic fields with depth.

The experiment is very simple to perform. When metal sheets are placed as shown the voltmeter connected to the "secondary" windings shows a decrease. The decrease should be proportional to the attenuation of the magnetic fields between the cores due to metal sheets. Therefore if the natural logarithm of the secondary voltages is plotted against the thickness of the pile of metal sheets, a "straight line" should be obtained from which the skin depth is easily determined.

Typical experimental results are shown in Fig. 2. The aluminum behaves as expected and yields a skin depth in reasonable agreement with the theoretical value of 11.7 mm at 50 Hz mains frequency. The iron shows a much smaller skin depth and some evidence of nonlinearity as expected because of permeability and hysteresis effects. Lead appears to give a value of skin depth much larger than the expected value of 3.3 cm. It has been suggested [Weaver (private communication)] that if the pile of lead sheets had thickness greater than the skin depth results probably would be improved. However the size of the apparatus would have to be expanded somewhat to accommodate these. Other metals could of course be used. Rather than taking conductivity values out of tables for the various metals, we used a 1-cm strip of the same sheet as a four-terminal resistor, passing the approximately 1-A primary current through it,

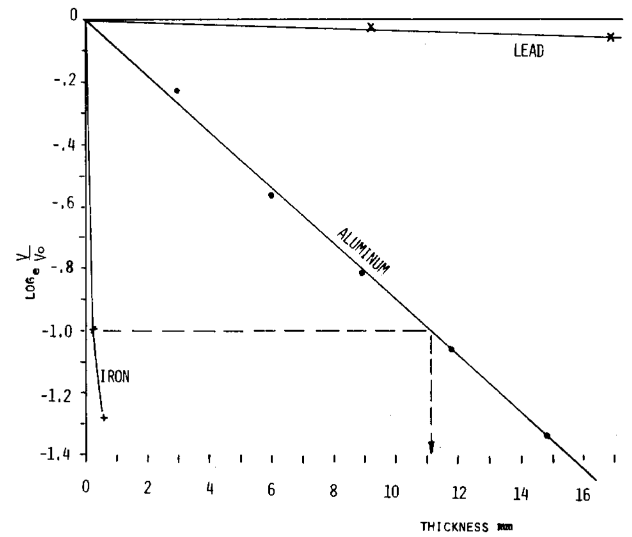

Fig. 2. Typical results of decrease of induced voltage (V) with depth for aluminum, iron, and lead. Dashed line shows the skin depth.

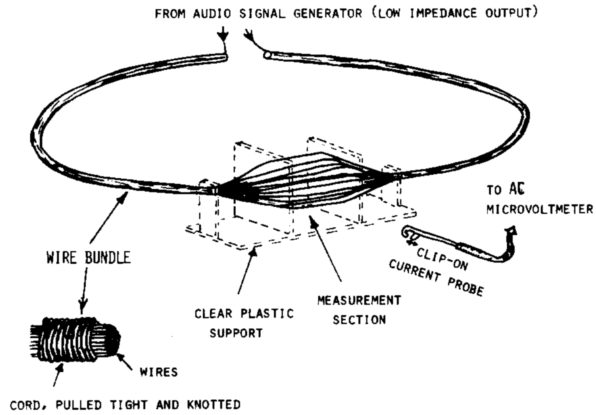

Fig. 3. Wire bundle apparatus for current-vs-depth measurements.

and measuring the voltage across the voltage terminals with the same ac microvoltmeter used in the second part of the experiment.

III. SKIN DEPTH EFFECT IN A WIRE

The obvious thing to do here it would seem is to measure the increased resistivity of a wire at high frequencies which results from the current being essentially confined to a surface layer. Two factors made this an unattractive experiment: (i) the deduction that the resistivity increase is due to the skin effect phenomenon is only indirect, and (ii) the experiment does not really work very conveniently as may be seen from the following considerations. If one uses a wire made from a good conductor, such as copper, the skin depth is given by $d = 66f^{-1/2}$ in mm, where f is the frequency in hertz. To get skin depths of a fraction of a mm, as would be required, it is necessary to use frequencies of the order of hundreds of kilohertz. The resistance of a short piece of wire would still be small and difficult to measure, and long pieces of wire would have inductive complications. The opposite extreme would be to make a wire out of a poor conductor, such as carbon, so that the resistances are higher and easier to measure. The skin depths are also larger so that one would have to use higher frequencies and the experiment again becomes impractical. Owing to these considerations we decided on a different approach which we believe is more successful in revealing what is happening.

Figure 3 shows the apparatus. What it consists of is a bundle of 109, No. 20 Standard Wire Gauge tinned copper wires which have been carefully packed so as to be in good electrical contact with one another over most of the wire bundle which has a total length of about 2½ m. The wires also were ordered so that the same wire lies in the same position throughout the bundle. In the center measurement section which should be kept short, the wires were insulated over a distance of about 50 cm with varnish and they diverge as shown and run between two clear plastic support brackets, still in the same relative position as in the bundle. They then converge into the bundle again. The wire bundle simulates a single large wire with the center section opened up so one can use a clip on current proble to measure the currents.

Figure 4 shows the cross section of the wire bundle. One hexagonal "ring" is shown shaded. These various rings were color coded in the measurement section using plastic sleeving and students sampled sufficient wires in each "ring" to get a representative current for that "ring" which was assumed to have a mean radius which could be obtained by scaling from Fig. 4.

The current probe was a unit built in our workshop utilizing a ½-in.-diam ferrite toroidal ring which was sawn in half and 150 turns of very fine copper wire wound on one section. The completed probe was found to have the following characteristic:

$$I_{\rm rms} = 5.64 \times 10^3\ V_{\rm rms} \div f \text{ (in hertz)},$$

where $V_{\rm rms}$ is the output voltage and $I_{\rm rms}$ the current in the wire around which the probe is clipped. Commercial current probes could equally well have been used here.

The current pattern as a function of depth is measured, in our experiment, at frequencies of 100, 1000, 10 000, and 50 000 Hz.

Typical experimental results are shown in Fig. 5 and theoretical results are shown as solid curves. The theory is given in Stratton.[1] The obvious discrepancy between theory

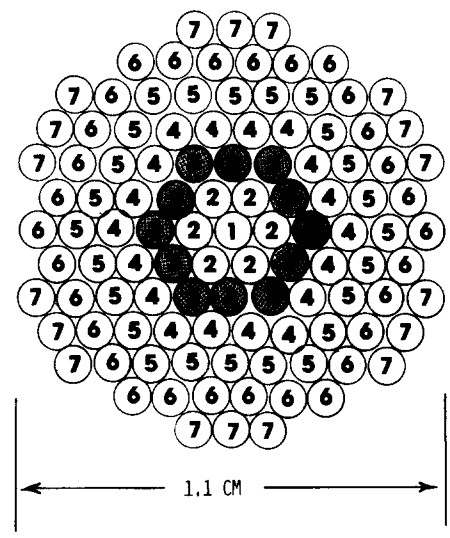

Fig. 4. Cross section of wire bundle. The 109, No. 20 Standard Wire Gauge copper wires are closely packed as shown. Measurements are taken as averages for the various "rings." Ring No. 3 is shown shaded.

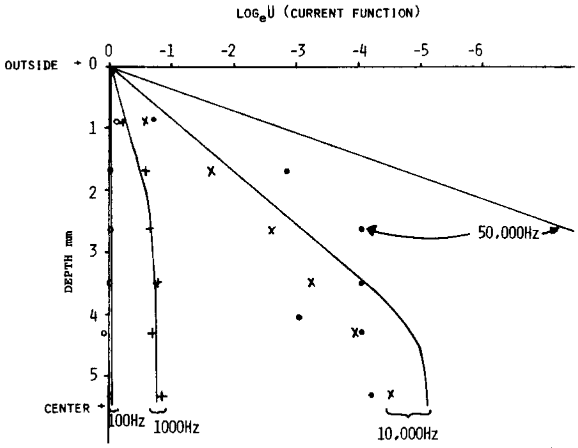

Fig. 5. Typical experimental results (individual points) compared with theoretical results (solid curves) for the wire bundle apparatus. Frequencies of 100 Hz (O), 1000 Hz (+), 10 000 Hz (X), and 50 000 Hz (·).

and experimental results at the higher frequencies is thought to be due to several reasons: (i) At the 50 000 Hz frequency the skin depth is smaller than the radii of the individual wires and the apparatus ceases to adequately simulate a single large wire. (ii) The outer ring No. 7 in Fig. 4 being incomplete means that there are large current differences between the exposed and the unexposed sections of ring No. 6 and hence to some extent in successive inner rings. It is felt that this tends to give current averages which are too large. It is recommended that ring No. 7 be omitted although this does change the geometry to a hexagonal configuration. (iii) Expanding the measurement section presumably has some adverse effect on the results.

In the actual experiment performed in our undergraduate laboratory these defects are not obviously revealed since the detailed comparison between exact theory and experiment is not attempted. The general form of decrease of current with depth is however shown; the decrease being about a factor of 100 between inside and outside at the higher frequencies. Near the surface the decrease for the higher frequencies should be closely that of a flat surface and the students can estimate the skin depth from their plots and compare with the theoretical, $(2/\mu\sigma\omega)^{1/2}$, depth. Unfortunately the defect (ii) above, tends here to give an undesired distortion to the curves in this particular bundle.

IV. CONCLUSIONS

The two apparatus described here illustrate the important basic aspects of the skin depth phenomenon. Resistivity increases can be deduced from the results obtained with the wire bundle apparatus.

As mentioned in the introduction there are other aspects of the skin effect phenomenon such as the association with eddy currents which have not been experimentally investigated here, and therefore these apparatus although they demonstrate the basic ideas about the skin effect which are usually included in elementary courses on electromagnetism, by no means demonstrate all the associated affects. As an example the skin effect in wires other than circular leads to more complex current distributions (see Terman[2]), and a modification of the bundle apparatus could be used there.

Finally, I should caution that constructing the wire bundle apparatus, although straightforward, is very time consuming.

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[1]J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941).

[2]F. E. Terman, *Electronic and Radio Engineering* (McGraw-Hill, New York, 1955).